BAULKHAM HILLS HIGH SCHOOL

2016

YEAR 12 TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks – 70

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 6 – 12

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

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1 Suppose θ is the acute angle between the lines $y - 2x = 3$ and $3y = -x + 2$. Which of the following is the value of $\tan\theta$?

(A) 7

(B) -7

(C) 1

(D) -1

2 Which of the following is an expression for $\int \sin^2 6x \, dx$?

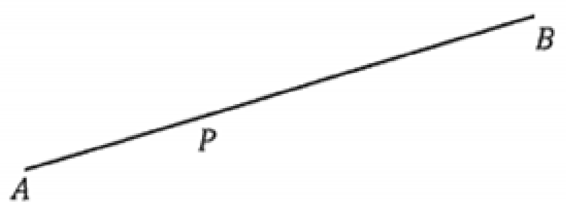
(A) $\frac{x}{2} - \frac{1}{12}\sin 6x + c$

(B) $\frac{x}{2} + \frac{1}{12}\sin 6x + c$

(C) $\frac{x}{2} - \frac{1}{24}\sin 12x + c$

(D) $\frac{x}{2} + \frac{1}{24}\sin 12x + c$

3 The point *P* divides the interval *AB* in the ratio 3:8



In what external ratio does *A* divide the interval *PB*?

(A) 11:3

(B) 8:3

(C) 3:5

(D) 3:11

4 The expression $\sin x - \sqrt{3}\cos x$ can be written in the form $2\sin(x + \alpha)$.
Find the value of α .

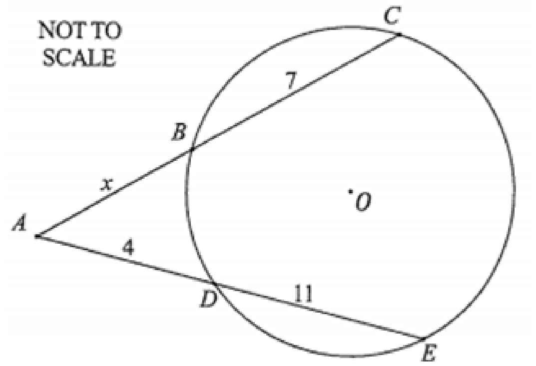
(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $-\frac{\pi}{3}$

(D) $-\frac{\pi}{6}$

5 In the diagram below, *BC* and *DE* are chords of a circle. *CB* and *ED* are produced to meet at *A*.



What is the value of *x*?

(A) 12

(B) 5

(C) $\frac{28}{11}$

(D) $\frac{11}{28}$

6 Four female and four male students are to be seated around a circular table. I how many ways can this be done if the males and females must alternate?

(A) $4! \times 4!$

(B) $3! \times 4!$

(C) $3! \times 3!$

(D) $2 \times 3! \times 4!$

7 It is known that $(x + 2)$ is a factor of the polynomial $P(x)$ and that

$$P(x) = (x^2 + x + 1) \times Q(x) + (2x + 3)$$

for some polynomial $Q(x)$. From this information alone, it can be deduced that;

(A) $Q(-2) = \frac{1}{3}$

(B) $Q(-2) = -\frac{1}{3}$

(C) $Q(-2) = 1$

(D) $Q(-2) = -1$

8 The velocity of a particle moving in simple harmonic motion in a straight line is given by $v^2 = 2 - x - x^2 \ \text{ms}^{-1}$, where x is displacement in metres.

Find the centre of the motion.

(A) $x = 2$

(B) $x = 1$

(C) $x = -\frac{1}{2}$

(D) $x = -2$

9 How many solutions does the equation $2x + 3\pi \sin x = 0$ have in the domain $0 \leq x \leq 2\pi$?

(A) 1

(B) 2

(C) 3

(D) 4

10 Which one of the following is the general solution of $2\sin^2\left(6t+\frac{\pi}{4}\right)=1$?

(A) $t=\frac{n\pi}{12}$, where n is an integer.

(B) $t=\frac{n\pi}{12}-\frac{\pi}{24}$, where n is an integer.

(C) $t=\frac{n\pi}{3}$ and $t=\frac{n\pi}{3}+\frac{\pi}{12}$, where n is an integer.

(D) $t=\frac{n\pi}{3}-\frac{\pi}{6}$ and $t=\frac{n\pi}{3}+\frac{\pi}{12}$, where n is an integer.

END OF SECTION I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour 45 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your BOS#. Extra paper is available.

In Questions 11 to 14, your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 11 ***(15 marks)*** Use a *separate* answer sheet

(a) Solve the inequality $\frac{1-x}{1+x} \leq 1$ *3*

(b) Dong and Kevin are two of ten candidates for a committee. *2*

How many ways can a committee of five be chosen, if Kevin refuses to be on the same committee as Dong?

(c) (i) Show that $\frac{\sin 2x}{1+\cos 2x} = \tan x$ *2*

(ii) Hence show that $\tan 15° + \cot 15° = 4$ *2*

(d) Evaluate $\int_0^3 \frac{x}{\sqrt{x+1}}\, dx$ using the substitution $x = u^2 - 1$ *3*

(e) A particle moves in a straight line along the x axis so that its acceleration is given by;

$$\ddot{x} = \frac{1}{4 + x^2}$$

where x is the displacement from the origin.

Initially the particle is at rest at the origin.

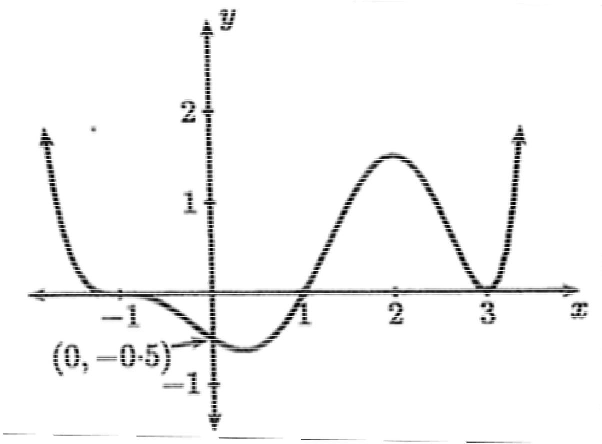
(i) Find v^2 as a function of x. *2*

(ii) Explain why v is always positive for $t > 0$ *1*

Marks

<u>Question 12</u> ***(15 marks)*** Use a *separate* answer sheet

(a) *2*



Write down a possible function $y = P(x)$ for the polynomial function sketched above. (Do NOT use calculus)

(b) Find $\int \frac{dx}{\sqrt{16 - 9x^2}}$ *2*

(c) Water is being heated in a kettle. At time t seconds, the temperature of the water is $T°$ C.

The rate of increase of the temperature of the water at any time after the kettle is switched on, is modelled by the equation $\frac{dT}{dt} = k(120 - T)$, where k is a positive constant.

The temperature of the water is at $20°$ C when the kettle is switched on.

(i) Show that $T = 120 - 100e^{-kt}$ is both a solution to the differential equation and satisfies the initial conditions. *2*

(ii) When the temperature of the water reaches $100°$ C, the kettle switches off. If it takes 10 seconds for the temperature to increase $10°$ C, once the kettle is switched on, find how long it takes for the kettle to switch off, to the nearest second. *2*

Question 12 continues on page 8

Marks

Question 12 ***(continued)***

(d) (i) Sketch the graph of the function $f(x) = e^x - 4$ showing clearly the coordinates of any points of intersection with the axes and the equations of any asymptotes. *1*

(ii) On the same diagram sketch the graph of the function $y = f^{-1}(x)$ showing clearly the coordinates of any points of intersection with the axes and the equations of any asymptotes. *1*

(iii) Find an expression for $y = f^{-1}(x)$ in terms of x *2*

(iv) Explain why the x coordinate of any point of intersections of the graphs $y = f(x)$ and $y = f^{-1}(x)$ satisfies the equation $e^x - x - 4 = 0$ *1*

(v) Taking $x_1 = -4$ as a first approximation, use one application of Newton's Method to find a second approximation for the point of intersection located in the third quadrant. Give your answer correct to two decimal places. *2*

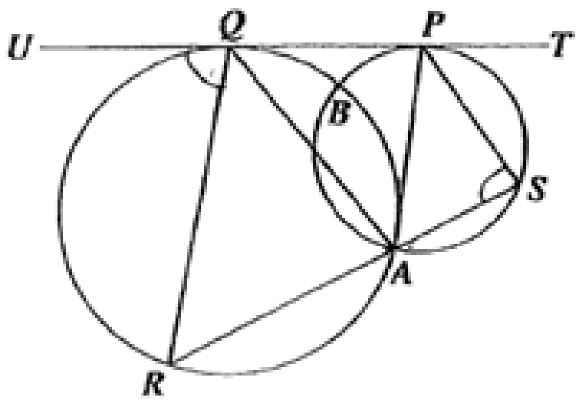
End of Question 12

Marks

Question 13 ***(15 marks)*** Use a *separate* answer sheet

(a) Use mathematical induction to prove that $3^{2n+4} - 2^{2n}$ is divisible by 5, for $n \geq 1$ *3*

(b) In the diagram below, two circles intersect at *A* and *B*. The common tangent *TU* touches the circles at *P* and *Q* respectively. A line through *A* cuts the left hand circle at *R* and the right hand circle at *S*, and it is found that *PQRS* is a cyclic quadrilateral.



Copy the diagram into your answer booklet.

(i) Give a reason why $\angle UQR = \angle PSA$ *1*

(ii) Prove that $PS || AQ$. *2*

(iii) Thus show that $\Delta PAS ||| \Delta QRA$ *2*

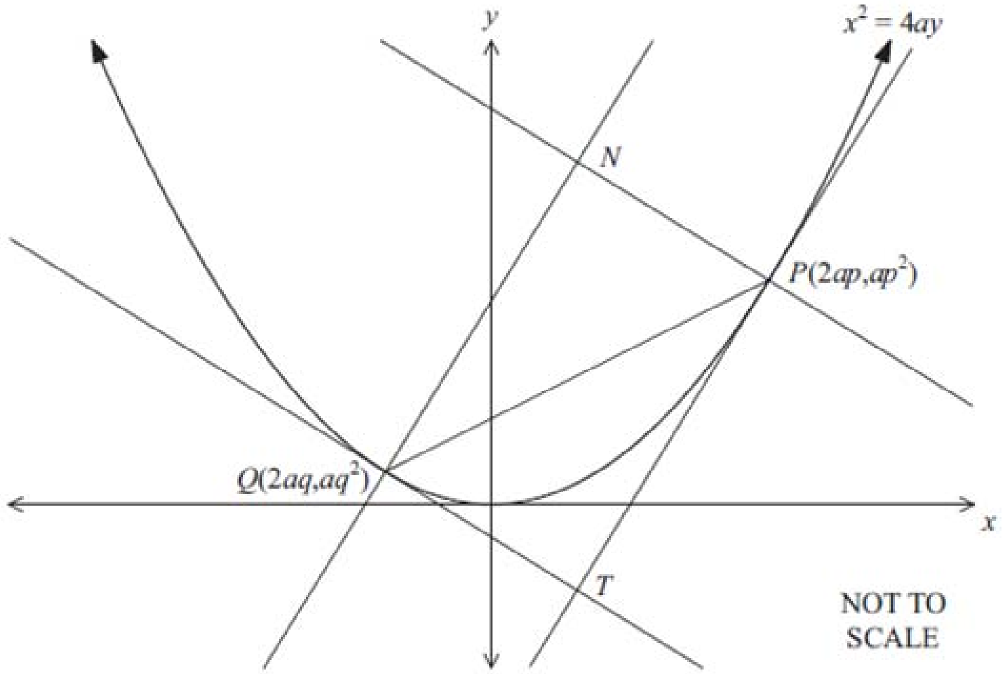
Question 13 continues on page 10

Marks

Question 13 ***(continued)***

(c) The diagram shows the parabola $x^2 = 4ay$. The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola such that PQ is a focal chord.

The tangents at P and Q intersect at T and the normal at P and Q intersect at N.



(i) Show that the equation of any chord PQ is given by $(p + q)x - 2y = 2apq$ *2*

(ii) Show that $pq = -1$ *1*

(iii) Explain why $PTQN$ is a cyclic quadrilateral *1*

(iv) Let C be the centre of the circle that passes through the points P, T,Q and N. Explain why C is the midpoint of PQ. *1*

(v) Find the Cartesian equation of the locus of C. *2*

End of Question 13

Marks

Question 14 ***(15 marks)*** Use a *separate* answer sheet

(a) (i) Neatly sketch the graph of $y = \sin^{-1}\left(\frac{x}{2}\right)$ clearly indicating the domain and range. *2*

(ii) By considering the graph in part (i), find the exact value of; *2*

$$\int_0^1 \sin^{-1}\left(\frac{x}{2}\right) dx$$

(b) A particle is moving in a straight line according to the equation;

$$x = 4\sin^2 t$$

where x is the displacement in metres and t is the time in seconds.

(i) Show that the particle moves in Simple Harmonic Motion. *2*

(ii) Find the value of x for which the speed is a maximum and determine that speed. *2*

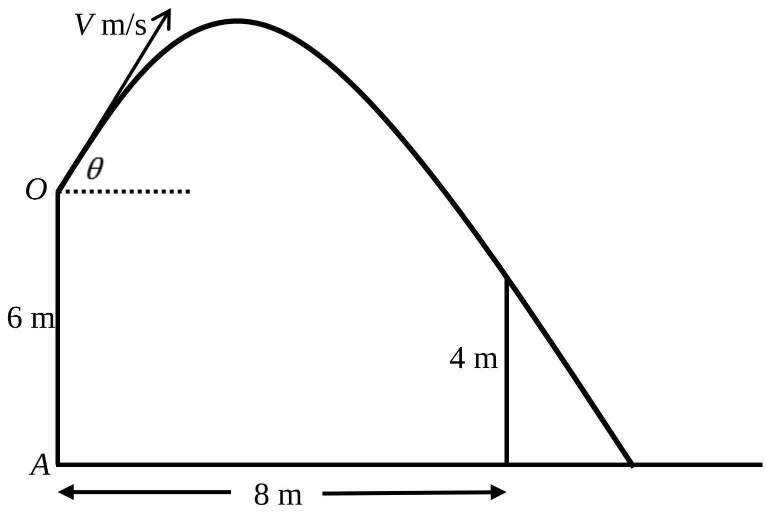
Question 14 continues on page 12

Marks

Question 14 (continued)

(c) A projectile is fired from a point *O*, which is 6 metres above horizontal ground, with initial velocity *V* m/s, at an angle of θ to the horizontal.

There is a thin vertical post which is 4 metres high and 8 metres horizontally away from a point *A*, directly below *O*, as shown in the diagram below.



The equations of motion are given by;

$$x = Vt\cos\theta \qquad \text{and} \qquad y = Vt\sin\theta - 4.9t^2$$

(Do NOT prove this)

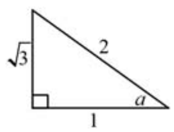
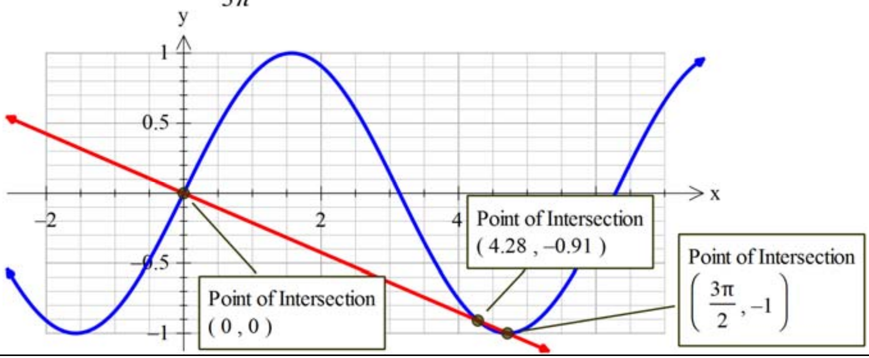
(i) If 2 seconds after projection, the projectile passes just above the top of the post, show that $\tan\theta = 2.2$ *2*

(ii) Show that the projectile hits the ground approximately 0.3 seconds after passing over the post. *3*

(iii) Find the angle that the projectile makes with the ground when it hits the ground, correct to the nearest degree. *2*

End of paper

BAULKHAM HILLS HIGH SCHOOL
YEAR 12 EXTENSION 1 TRIAL 2016 SOLUTIONS

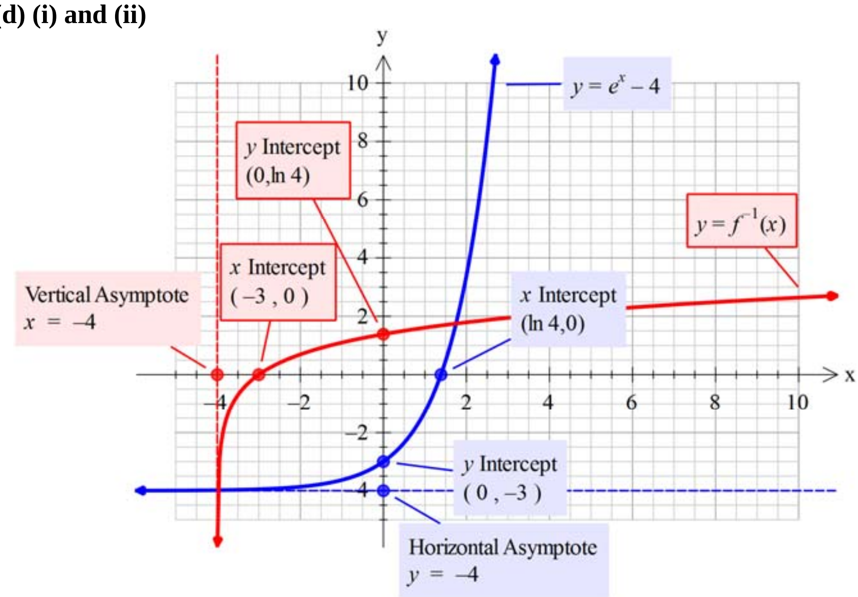
Solution	Marks	Comments
SECTION I		
1. A – $y-2x=3 \Rightarrow m_1=2$ $\quad 3y=-x+2 \Rightarrow m_2=-\frac{1}{3}$ $\tan\theta=\left\lvert\frac{2+\frac{1}{3}}{1+(2)\left(-\frac{1}{3}\right)}\right\rvert$ $=\left\lvert\frac{\frac{7}{3}}{\frac{1}{3}}\right\rvert$ $=7$	1	
2. C – $\int \sin^2 6x\,dx=\frac{1}{2}\int(1-\cos 12x)dx$ $=\frac{1}{2}\left(x-\frac{1}{12}\sin 12x\right)+c$ $=\frac{x}{2}-\frac{1}{24}\sin 12x+c$	1	
3. D - $AP=3x$, $BP=8x$ $\therefore \frac{AP}{AB}=\frac{3x}{11x}$ $=\frac{3}{11}$	1	
4. C – $\sin x-\sqrt{3}\cos x=2\sin\left(x-\frac{\pi}{3}\right)$ $\quad \tan a=\sqrt{3}$ $=2\sin(x+\alpha)$ $\quad \alpha=\frac{\pi}{3}$  $\therefore \alpha=-\frac{\pi}{3}$	1	
5. B - $AB\times AC=AD\times AE$ (product of intercepts of intersecting secants) $x(x+7)=4\times 15$ $x^2+7x-60=0$ $(x+12)(x-5)=0$ $x=-12$ or $x=5$ $\quad$ but $x>0$ $\therefore x=5$	1	
6. B – Ways $=3!\times 4!$	1	
7. A - $P(-2)=0$ $\left[(-2)^2-2+1\right]Q(-2)+[2(-2)+3]=0$ $3Q(-2)-1=0$ $Q(-2)=\frac{1}{3}$	1	
8. C – $v^2\geq 0$ $2-x-x^2\geq 0$ $x^2+x-2\leq 0$ $(x+2)(x-1)\leq 0$ $-2\leq x\leq 1$ $\quad \therefore$ centre is $x=-\frac{1}{2}$	1	
9. C - $2x+3\pi\sin x=0$ $\quad$ From the graph, there are three points of intersection $\sin x=-\frac{2}{3\pi}x$  	1	

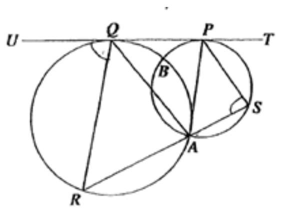
Solution	Marks	Comments
10. A - $2\sin^2\left(6t+\frac{\pi}{4}\right)=1$ $1-2\sin^2\left(6t+\frac{\pi}{4}\right)=0$ $\cos\left(12t+\frac{\pi}{2}\right)=0$ $-\cos\left(\frac{\pi}{2}-12t\right)=0$ $\sin 12t=0$ $12t=n\pi+(-1)^n\sin^{-1}0$ where n is an integer $12t=n\pi$ $t=\frac{n\pi}{12}$	1	

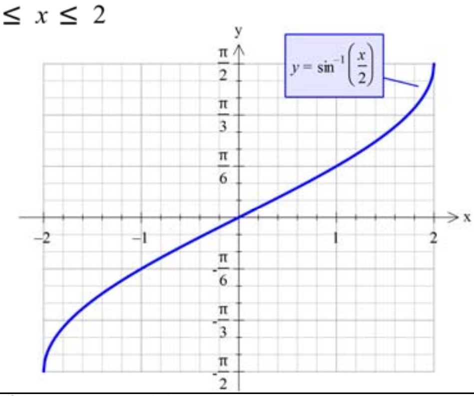
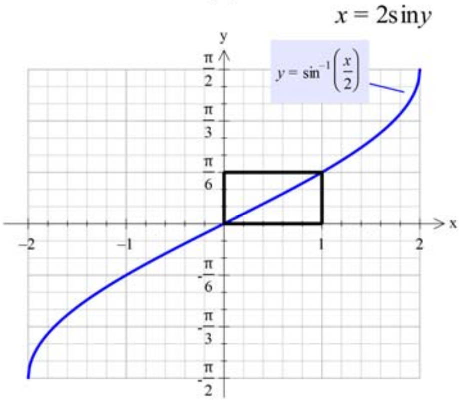
SECTION II

Solution	Marks	Comments
QUESTION 11		
11(a) $\frac{1-x}{1+x}\leq 1$ $1+x\neq 0$ $x\neq -1$ $1-x=1+x$ $2x=0$ $x=0$  $x<-1$ or $x\geq 0$	3	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation , with no consideration of the denominator.
11(b) Committees $={}^{10}C_5-{}^{8}C_5$ $=252-56$ $=196$	2	**2 marks** • Correct solution **1 mark** • Identifies logical cases that must be considered • Successfully finds one of the required cases
11(c) (i) $\frac{\sin 2x}{1+\cos 2x}=\frac{2\sin x\cos x}{1+2\cos^2 x-1}$ $=\frac{2\sin x\cos x}{2\cos^2 x}$ $=\frac{\sin x}{\cos x}$ $=\tan x$	2	**2 marks** • Correct solution **1 mark** • Correctly uses **both** $\sin 2\theta$ and $\cos 2\theta$ results
11 (c) (ii) $\tan 15°=\frac{\sin 30°}{1+\cos 30°}$ $=\frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$ $=\frac{1}{2+\sqrt{3}}$ $\tan 15°+\cot 15°=\frac{1}{2+\sqrt{3}}+\frac{2+\sqrt{3}}{1}$ $=\frac{1+(2+\sqrt{3})^2}{2+\sqrt{3}}$ $=\frac{1+4+4\sqrt{3}+3}{2+\sqrt{3}}$ $=\frac{8+4\sqrt{3}}{2+\sqrt{3}}$ $=4$	2	**2 marks** • Correct solution **1 mark** • Uses given result to find $\tan 15°$

Solution	Marks	Comments
11 (d) $\int_0^3 \frac{x}{\sqrt{x+1}}\,dx = \int_1^2 \frac{u^2-1}{u} \times 2u\,du$ $x = u^2 - 1$, $dx = 2u\,du$ when $x = 0, u = 1$; when $x = 3, u = 2$ $= 2\int_1^2 (u^2 - 1)\,du$ $= 2\left[\frac{1}{3}u^3 - u\right]_1^2$ $= 2\left(\frac{8}{3} - 2 - \frac{1}{3} + 1\right)$ $= \frac{8}{3}$	**3**	**3 marks** • Correct solution using the given substitution • *Note: solving as an indefinite integral, then using answer to find definite integral is acceptable* **2 marks** • Correct primitive in terms of *u* • Correct integrand in terms of *u*, including the correct limits **1 mark** • Correct integrand in terms of *u* without the limits • Correctly finds answer using an alternative approach
11 (e) (i) $v\frac{dv}{dx} = \frac{1}{4+x^2}$ $\int_0^v v\,dv = \int_0^x \frac{dx}{4+x^2}$ $\frac{1}{2}[v^2]_0^v = \frac{1}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^x$ $v^2 = \tan^{-1}\left(\frac{x}{2}\right)$	**2**	**2 marks** • Correct solution **1 mark** • Correctly uses the idea that $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ or equivalent
11 (e) (ii) $\ddot{x} = \frac{1}{4+x^2} > 0$ for all values of x Since particle begins at rest, positive acceleration means the particle moves in the positive direction, and will not slow down unless acceleration becomes negative. As this never occurs, $v > 0$	**1**	**1 mark** • Correct explanation
QUESTION 12		
12 (a) By examining the zeros in the sketch, the polynomial must take the form $y = k(x+1)^3(x-1)(x-3)^2$ y intercept is $-\frac{1}{2}$ $\therefore a(1)(-1)(9) = -\frac{1}{2}$ $-9a = -\frac{1}{2}$ $a = \frac{1}{18}$ $\therefore$ a possible polynomial is $y = \frac{1}{18}(x+1)^3(x-1)(x-3)^2$	**2**	**2 marks** • Correct solution **1 mark** • Uses *y* intercept in a valid attempt to find function • Identifies the nature of the zeros
12(b) $\int \frac{dx}{\sqrt{16-9x^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{\frac{16}{9} - x^2}}$ $= \frac{1}{3}\sin^{-1}\left(\frac{x}{\frac{4}{3}}\right) + c$ $= \frac{1}{3}\sin^{-1}\frac{3x}{4} + c$	**2**	**2 marks** • Correct solution **1 mark** • Uses correct standard integral
12 (c) (i) $T = 120 - 100e^{-kt}$ $\frac{dT}{dt} = 100ke^{-kt}$ $= k(100e^{-kt} - 120 + 120)$ $= k(120 - T)$ when $t = 0$, $T = 120 - 100e^0$ $= 120 - 100$ $= 20$	**2**	**2 marks** • Correct solution **1 mark** • Establishes initial temperature is 20° • Verifies given equation is solution to the differential equation

Solution	Marks	Comments
12 (c) (ii) when $t = 10$ $T = 30$ $30 = 120 - 100e^{-10k}$ $e^{-10k} = \frac{9}{10}$ $-10k = \ln\frac{9}{10}$ $k = -\frac{1}{10}\ln\frac{9}{10}$ $= 0.01053605157...$ when $T = 100$, $100 = 120 - 100e^{-kt}$ $100e^{-kt} = 20$ $e^{-kt} = \frac{1}{5}$ $-kt = \ln\frac{1}{5}$ $t = -\frac{1}{k}\ln\frac{1}{5}$ $t = 152.7553185...$ The kettle switches off after 153 seconds	**2**	**2 marks** • Correct solution **1 mark** • Finds correct value of k • Uses their value of k correctly to find a solution *Note: no rounding penalty*
12 (d) (i) and (ii)  	**1**	**(i)** **1 mark** • Correct graph with all features labelled
	1	**(ii)** **1 mark** • Correct graph with all features labelled
12 (d) (iii) $f^{-1}: x = e^y - 4$ $e^y = x + 4$ $y = \ln(x + 4)$	**2**	**2 marks** • Correct solution **1 mark** • Finds an expression in terms of y
12 (d) (iv) The two curves intersect on the line $y = x$ $\therefore\ x = e^x - 4$ $e^x - x - 4 = 0$	**1**	**1 mark** • Correct explanation
12 (d) (v) $x_1 = x_0 - \frac{fx_0}{f'(x_0)}$ $= x_0 - \frac{e^x - x - 4}{e^x - 1}$ $= -4 - \frac{e^{-4} + 4 - 4}{e^{-4} - 1}$ $= -3.98134...$ $= -3.98$ (correct to 3 decimal places)	**2**	**2 marks** • Correct solution **1 mark** • Uses Newton's Method correctly

Solution	Marks	Comments
QUESTION 13		
13 (a) When $n = 1$; $3^{(2(1)+4)} - 2^{2(1)} = 3^6 - 2^2$ $= 729 - 4$ $= 725$ which is divisible by 5 Hence the result is true for $n = 1$ Assume the result is true for $n = k$ i.e. $3^{2k+4} - 2^{2k} = 5P$ where P is an integer Prove the result is true for $n = k + 1$ i.e. Prove $3^{2k+6} - 2^{2k+2} = 5Q$ where Q is an integer ***PROOF:*** $3^{2k+6} - 2^{2k+2} = 3^2(3^{2k+4}) - 2^{2k+2}$ $= 9(5P + 2^{2k}) - 2^2(2^{2k})$ (by assumption) $= 45P + 9 \times 2^{2k} - 4 \times 2^{2k}$ $= 45P + 5 \times 2^{2k}$ $= 5(9P + 2^{2k})$ $= 5Q$ where $Q = 9P + 2^{2k}$ which is an integer Hence the result is true for $n = k + 1$,if it is true for $n = k$ Since the result is true for $n = 1$, then it is true for all positive integers by induction.	3	There are 4 key parts of the induction; 1. Proving the result true for $n = 1$ 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof 4. Correctly proving the required statement **3 marks** • Successfully does all of the 4 key parts **2 marks** • Successfully does 3 of the 4 key parts **1 mark** • Successfully does 2 of the 4 key parts
13 (b) (i) exterior angle of a cyclic quadrilateral equals the opposite interior angle	1	**1 mark** • Correct reason
13 (b) (ii) $\angle UQR = \angle QAR$ (alternate segment theorem) $\therefore \angle PSA = \angle QAR$ (explained in (i)) Thus $PS \parallel AQ$ as the corresponding angles are equal 	2	**2 marks** • Correct proof **1 mark** • Uses alternate segment theorem or similar in a valid approach
13 (b) (iii) $\therefore \angle PSA = \angle QAR$ (explained in (i)) $PA \parallel QR$ (similar method to part(ii) – commencing with $\angle TPS$) $\angle PAS = \angle QRA$ (corresponding $\angle$'s, $PA \parallel QR$) $\Delta PAS \parallel\mid \Delta QRA$ (AA)	2	**2 marks** • Correct proof **1 mark** • Uses properties of parallel lines or similar in a valid approach
13 (c) (i) $m_{PQ} = \dfrac{ap^2 - aq^2}{2ap - 2aq}$ $= \dfrac{a(p+q)(p-q)}{2a(p-q)}$ $= \dfrac{p+q}{2}$ $y - ap^2 = \dfrac{(p+q)}{2}(x - 2ap)$ $2y - 2ap^2 = (p+q)x - 2ap^2 - 2apq$ $(p+q)x - 2y = 2apq$	2	**2 marks** • Correct proof **1 mark** • Find the slope of the chord PQ
13 (c) (ii) focal chord passes through focus $(0,a)$ $(0,a) : (p+q)(0) - 2a = 2apq$ $-2a = 2apq$ $pq = -1$	1	**1 mark** • Correctly shows result
13 (c) (iii) Tangents and normals are perpendicular to each other $\therefore \angle NQT = \angle NPT = 90°$ Thus $PTQN$ is a cyclic quadrilateral as a pair of opposite angles are supplementary	1	**1 mark** • Correctly explanation
13(c) (iv) $4ay = x^2$ $y = \dfrac{x^2}{4a}$ $\dfrac{dy}{dx} = \dfrac{x}{2a}$ when $x = 2ap$ $\dfrac{dy}{dx} = p$ Thus the slope of PT is p, and the slope of QT is q. As $pq = -1$, $PT \perp QT$ PQ is the diameter of the circle ($\angle$ in semicircle) Consequently C must be the midpoint of the diameter PQ	1	**1 mark** • Correctly explanation • *Note: it is not necessary to derive the slope of the tangent.*

Solution	Marks	Comments
13 (c) (v) $x_C = \frac{2ap+2aq}{2}$ $= a(p+q) \Rightarrow p+q = \frac{x}{a}$ $y_C = \frac{ap^2+aq^2}{2}$ $= \frac{a(p^2+q^2)}{2}$ $= \frac{a\left[(p+q)^2-2pq\right]}{2}$ $= \frac{a}{2}\left[\left(\frac{x}{a}\right)^2+2\right]$ as $pq=-1$ $= \frac{x^2}{2a}+a$ ∴ locus is $y = \frac{x^2}{2a}+a$ OR $2a(y-a)=x^2$	2	**2 marks** • Correct solution **1 mark** • Find the coordinates of *C*
QUESTION 14		
14 (a) (i) domain: $-1 \le \frac{x}{2} \le 1$ range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ $-2 \le x \le 2$ 	2	**2 marks** • Correct graph with domain and range indicated **1 mark** • Identifies both domain and range • Correct curve
14 (a) (ii) $y = \sin^{-1}\left(\frac{x}{2}\right) \Rightarrow \sin y = \frac{x}{2}$ $x = 2\sin y$  $\int_0^1 \sin^{-1}\left(\frac{x}{2}\right) dx = \frac{\pi}{6} \times 1 - 2\int_0^{\frac{\pi}{6}} \sin y \, dy$ $= \frac{\pi}{6} + 2[\cos y]_0^{\frac{\pi}{6}}$ $= \frac{\pi}{6} + 2\left(\frac{\sqrt{3}}{2} - 1\right)$ $= \frac{\pi}{6} + \sqrt{3} - 2$	2	**2 marks** • Correct solution **1 mark** • Correct answer obtained without referencing the graph • Uses the graph in a valid attempt to find the solution
14 (b) (i) $x = 4\sin^2 t$ $\dot{x} = 8\sin t\cos t$ $= 4\sin 2t$ $\ddot{x} = 8\cos 2t$ $= 8 - 16\sin^2 t$ $= -4(4\sin^2 t - 2)$ $= -4(x-2)$ Hence the particle moves in *SHM* as $\ddot{x} = -4X$ where $X = x - 2$	2	**2 marks** • Correct solution **1 mark** • Establishes a motion equation for acceleration • Recognises the condition for a particle to be moving in SHM
14 (b) (ii) maximum speed occurs at the centre of motion i.e. $x = 2$ $\dot{x} = 4\sin 2t$ Thus the maximum speed is 4 m/s	2	**2 marks** • Correct solution **1 mark** • Finds the centre of motion • Finds speed at the calculated centre of motion *Note: do not penalise for answers obtained from non-trivialised solutions to part (i)*

Solution	Marks	Comments
14 (c) (i) When $t = 2$; $x = 8$, $8 = 2V\cos\theta$, $V\cos\theta = 4$; $y = -2$, $-2 = 2V\sin\theta - 19.6$, $V\sin\theta = 8.8$ $\frac{V\sin\theta}{V\cos\theta} = \frac{8.8}{4}$ $\tan\theta = 2.2$	**2**	**2 marks** • Correct solution **1 mark** • Uses $y = 4$ and concludes $\tan\theta = 2.95$
14 (c) (ii) Projectile hits the ground when $y = -6$ $-6 = VT\sin\theta - 4.9T^2$ however $V\cos\theta = 4 \Rightarrow V = \frac{4}{\cos\theta}$ $-6 = \frac{4T\sin\theta}{\cos\theta} - 4.9T^2$ $4.9T^2 - 8.8T - 6 = 0$ $T = \frac{8.8 \pm \sqrt{8.8^2 + 4(6)(4.9)}}{9.8}$, but $T > 0$ $T = \frac{8.8 + \sqrt{195.04}}{9.8}$ $= 2.3230...$ ∴ the projectile hits the ground approximately 0.3 seconds after passing the pole	**3**	**3 marks** • Correct solution **2 marks** • Finds the cartesian equation of motion that can be used to solve the problem, or equivalent merit **1 mark** • Attempts to eliminate V from the motion equations • Finds a quadratic equation of motion.
14 (c) (iii) $\dot{x} = V\cos\theta$ $= 4$ $\dot{y} = V\sin\theta - 9.8T$ $= 8.8 - 9.8(2.3)$ $= -13.74$ $\tan\alpha = \frac{\dot{y}}{\dot{x}}$ $= -\frac{13.74}{4}$ $\alpha = 73.76862255...$ ∴ the projectile makes an angle of $74°$ when striking the ground	**2**	**2 marks** • Correct solution **1 mark** • Finds the horizontal and vertical components of velocity • Finds one component and uses $\frac{\dot{y}}{\dot{x}}$